Greenhouse Gas Accumulation and Demand-driven Economic Growth – A Simulation Model

Lance Taylor, Duncan K. Foley, Jonathan F. Cogliano, and Rishabh Kumar

Abstract: A demand-driven macroeconomic model incorporating damages from greenhouse gas accumulation is presented. Comparative statics are analyzed and the model is calibrated numerically on a world economy basis. It is extended to a system of differential equations describing changes over time in the capital/population ratio, atmospheric CO₂ concentration, and levels of labor and energy productivity as they respond to growth of fossil fuel energy use. Strong cyclicality in capital per capita (a “prey” variable) and CO₂ concentration (a “predator”) is observed, which leads to economic collapse seven or eight decades into a “business as usual” simulation. It can be avoided by expenditure of roughly one-half of worldwide defense outlays on mitigating CO₂ emission. Further comparative dynamics of the mitigated system are discussed.

Key words: Demand-driven growth, climate change, simulation

JEL classifications: E12, E2, Q54

* Revised version of a paper presented at a conference on “Achieving 2°C climate stabilization: macroeconomic benefits or costs?” sponsored by The Cambridge Trust for New Thinking in Economics, Cambridge UK, 4-5 July 2013. Taylor, Foley, and Kumar are at the New School for Social Research; Cogliano is at Dickinson College. Research supported by the Institute for New Economic Thinking (INET) under a grant to the Schwartz Center for Economic Policy Analysis at the New School. Contributions by Armon Rezai and Gregor Semieniuk and comments by conference participants are gratefully acknowledged.
This paper presents a long-run numerical simulation model of the interactions between greenhouse gas (GHG) accumulation and economic growth. The specification contains several novel features.

Output, employment, and capital stock growth are set by aggregate demand, in contrast to the supply side full employment assumptions in most other analyses of linkages between the macroeconomy and climate change. In particular, “optimal growth” modeling is replaced by interactions between demand and the income distribution.

In detail, distribution between real wages and profits depends on the employment/population ratio as determined by the capital stock per capita, the output/capital ratio (or “capital utilization”), and the output/labor ratio (or labor productivity). Consistent with contemporary time series data, the profit share falls when employment rises, in turn cutting investment and effective demand.

Higher GHG concentration in the atmosphere has potential negative effects on profits and investment and the growth of the capital stock. Labor productivity growth could also be reduced but an alternative hypothesis is that it might increase as economic actors adjust to adverse climatic change. Both possibilities are addressed in model simulations. As suggested by ecological economics, labor productivity growth responds positively to fossil fuel “energy intensity” or the ratio of energy in use to employment. Energy productivity or the ratio of output to energy follows from labor productivity and energy intensity as a matter of definition.

GHG accumulation is driven by energy use. It responds negatively to the level of expenditure on mitigation of CO₂ emission, which also stimulates effective demand. To simplify the specification, it is assumed that the atmospheric concentration of CO₂
directly influences the economic system rather than operating through rising global temperature. As will be seen, growth in the capital stock per capita turns negative roughly 70 years into a “business as usual” (BAU) simulation when CO₂ concentration approaches 600 parts per million by volume (ppmv), a level typically associated with an increase of global temperature of two to three degrees Celsius. Following this shift, output and employment fall dramatically before approaching very low steady state levels two centuries into the simulation. Mitigation spending on the order of 1.25% of output shifts the dynamics. It offsets the capital stock oscillation and permits stable output growth with slowly rising GHG concentration (see Figure 6 below).

The BAU climate crisis reflects an important aspect of the model: it incorporates long-term cyclical interactions between the capital/population ratio and the level of GHG in the atmosphere. That is, higher economic activity (a “prey” variable) generates faster GHG accumulation, but more GHG (as a “predator”) cuts into output. This feedback loop is suppressed in optimizing models of supply-driven economic growth and greenhouse gas which typically converge monotonically to a steady-state growth path.

In what follows, the model is set out in equation form. Simulations follow, concluding with a more neoclassical specification which also displays cyclicality. An appendix outlines data and parameterization for the simulations.

The Model

The specification incorporates output and investment determination, income distribution between profits and wages and demand, labor productivity and energy intensity, capital stock growth, energy use and GHG accumulation, and dynamics of population and energy intensity.
Output and Investment

We begin with the level of economic activity in the short run as a prelude to the analysis of long-term growth and distribution. Let $X$ be real output; $C$, consumption; $I$, investment (including inventory accumulation and gross fixed capital formation); $M$, expenditure on GHG mitigation; and $H$, fiscal spending (aside from mitigation). All these variables are “flows” (trillions of dollars) per year. In the simulations below, it is convenient to scale mitigation to output, $M = mX$. Investment and government spending can be scaled to the capital stock $K$, $I = gK$ and $H = hK$ respectively. Let $\tau$ be the tax rate on output.¹ As discussed below, $K$ is simply accumulated investment less depreciation. In contrast to neoclassical models it plays no direct productive role (there is no “aggregate production function”), but does determine the scale of the economic system.

Consumption will be $C = [1 - s(\pi) - \tau]X$ in which $s(\pi)$ is the saving rate treated as an increasing function of the share $\pi$ of profits in total income (in turn equal to output). With $s_\pi$ and $s_w$ as shares of profit and wage income that are saved, a convenient expression for the overall saving rate is $s(\pi) = s_w + (s_\pi - s_w)\pi$. Clearly, $ds/d\pi > 0$ for $s_\pi > s_w$. This difference between saving rates from the two sources of income is widely observed in the data.

The macro balance equation is

$$X = C + I + H + M = [1 - s(\pi) - \tau]X + I + H + M.$$  

¹ Many theoretical demand-driven growth models do not include fiscal spending and taxation. But to fit national accounts data and generate plausible multipliers and dynamics it is essential to carry them in the specification.
Along lines proposed by Kalecki (1971) assume that gross fixed capital formation is driven by the profit rate $r = \pi u$ with $u = X/K$ ($u$ stands for utilization)

$$I = (g_0 + \alpha \pi u)K.$$  

Macro balance becomes

$$X = [1 - s(\pi) - \tau]X + (g_0 + \alpha \pi u)K + hK + mX$$

or

$$[s(\pi) - \alpha \pi + \tau - m]u = g_0 + h \quad .$$  

This expression can also be written as

$$s(\pi)u = g + (h - \tau u) + mu$$

saying that saving finances investment, the fiscal deficit, and mitigation.

Reflecting capital’s role in scaling the macro system, $X$ is proportional to $K$ in these equations, i.e. for a given level of $\pi$ the elasticity of $X$ with respect to $K$ is equal to one. This relationship will change below when endogenous shifts in $\pi$ are brought into the picture. From (1) utilization is determined by an effective demand relationship

$$u(\pi) = [s(\pi) - \alpha \pi + \tau - m]^{-1}(g_0 + h) \quad .$$  

Because $\pi$ enters the first term on the right both positively and negatively, the sign of the partial derivative $\partial u / \partial \pi$ is ambiguous. In line with international evidence (Kiefer and Rada, 2013) we assume that $\partial u / \partial \pi > 0$, or output is “profit-led.” The $u(\pi)$ schedule slopes upward in Figure 1 (based on the parameterization outlined in the appendix).  

**Figure 1**

---

2 Equation (2) differs from the standard multiplier expression $u = (s + \tau)^{-1}(g_0 + h)$ because it includes the terms $-\alpha \pi$ and $-m$. The former reflects the feedback of higher economic activity into investment demand and the latter the demand-stimulating role of mitigation spending. A formula like (2) is sometimes said to incorporate a “super-multiplier.”
A reduced form gross investment schedule can be written as

\[
g(\pi) = [s(\pi) - \alpha \pi + \tau - m]^{-1}[(s(\pi) + \tau - m)g_0 + \alpha \pi h],
\]

again depending on \( K \) only through changes in \( \pi \).

From the numbers in the appendix, the “base” output multiplier is \([s(\pi) - \alpha \pi + \tau]^{-1} = 2.2727\). This multiplier is on the high side, but will be modified when change in \( \pi \) are brought into consideration. Also, there is no demand leakage due to imports because Earth does not trade with other planets.

Distribution

Supply-driven climate models typically incorporate a “damage function” which indicates the degree to which output at full employment is directly reduced by an increase in CO₂ concentration, currently on the order of 400 ppmv. Because its output is determined by demand, the present model cannot admit such a simple relationship. To find the level of capacity utilization, one has to bring in responses of the profit share \( \pi \) to changes in \( u, G, \) the capital/population ratio \( \kappa \), and labor productivity \( \xi = X/L \) with \( L \) as employment.

Through changes in effective demand, \( u \) and \( g \) in turn will respond to these variables. Besides a direct negative impact of \( G \) on \( \pi \), higher GHG concentration could affect productivity as discussed below. It could also destroy capital, either directly (say a Category V hurricane hits the Houston ship channel and wipes out 10% of US refining capacity) or via more rapid depreciation.

To trace through these possibilities we have to set up a profit share schedule \( \pi(u) \) to use together with the \( u(\pi) \) curve in Figure 1. Ignoring complications arising from
shifts in labor force participation rates, let \( N \) be the total population. Then the employment rate is \( \lambda = L/N \). Also \( \kappa = K/N \) is the capital stock per capita. A convenient identity links these variables with economic activity \( u \),

\[
\lambda = \kappa u / \xi .
\]

At any time, \( \kappa \) will be determined by the history of capital accumulation and population growth. Productivity \( \xi \) can be assumed to follow from a “technical progress function” discussed below. In (4) \( \lambda \) becomes proportional to \( u \).

In several passages in *Capital*, Marx sketched a theory of business cycles (later formalized by Goodwin, 1967) pivoting on shifts in the income distribution. At the bottom of a cycle, the real wage is held down by a large reserve army of un- or under-employed workers, and capitalists can accumulate freely. However, as output expands the reserve army is depleted and \( \lambda \) goes up. The real wage rises in response to a tighter labor market, forcing a profit squeeze. Capitalists search for new labor-saving technologies and also invest to build up the stock of capital and reduce employment via input substitution. Excessive funds tied up in machinery, sectoral imbalances, and lack of purchasing power on the part of capitalists to sustain investment (or on the part of workers to absorb the output that new investment produces) can all underlie a cyclical collapse. The model of this paper extends this cyclical dynamics toward the long run, with GHG accumulation triggering the collapse.

A simple formulation is that a higher level of \( \lambda \) reduces \( \pi \) in an equation such as

\[
\pi = \phi(\lambda, G) = \phi(\kappa u / \xi, G).
\]

---

3 In the simulations below, the level of population is treated as a function of time, independently of the rest of the model.
Before getting into a detailed specification, note that $\partial \phi / \partial \lambda$ and $\partial \phi / \partial G$ are both negative. The effect of $u$ on $\pi$ via $\lambda$ is illustrated as the $\pi(u)$ schedule in Figure 1.\footnote{In standard national accounting, output $X$ would be measured as real value-added. Hence a lower $\pi$ would have to be met by higher labor share $\psi = 1 - \pi = \omega / \xi$ with $\omega$ as the real wage. It is assumed below that energy use $E$ is proportional to output, $\varepsilon = X / E$, with $\varepsilon$ as energy productivity. The quantity $Y = (1 + \varepsilon^{-1})X$ would be a natural definition of “gross” real output, and a change in $\pi$ would normally be reflected into changes in the shares of labor and the total cost of energy $P_{e}E$ with $P_{e}$ as the price of energy in a larger value aggregate corresponding to $Y$. For simplicity this accounting detail is omitted here.}

Now consider the effects of possible changes. For a given $u$, lower labor productivity $\xi$ (treated for the moment as exogenous) will increase $\lambda$ and make the profit share fall. After the $\pi(u)$ curve shifts downward, both $\pi$ and $u$ would decline. A similar outcome occurs via the direct negative effect of higher $G$ on $\pi$ – the demand-driven analog of the GHG damage function in mainstream models. The rationale is that higher GHG concentration will raise the costs of doing business through various channels – falling crop yields, bigger outlays on maintenance and repair, disruptions to supply chains and inventory management, etc. – thereby cutting into profitability and effective demand.

A lower level of $\kappa$ (capital destruction) will reduce $\lambda$ and increase both $\pi$ and $u$ as the $\pi(u)$ schedule shifts up. The implication of the higher $u$ is that output and employment decrease less than in proportion to the capital stock while the profit share rises.\footnote{In terms of the Houston ship channel example mentioned above, $\pi$ would presumably rise due to higher gasoline prices, stimulating investment demand to help offset the destruction of capital.} In other words, the elasticity of $X$ with respect to $K$ lies between zero and one (more on this response below). Higher spending on mitigation would shift the $u(\pi)$ schedule toward the right, driving economic activity and GHG emission up.
accompanied by a lower profit share. The output multiplier would be lower, the steeper the $\pi(u)$ schedule.

In simulations we used specific functional forms to capture the effects of $G$ and $\lambda$ on $\pi$,

$$\pi(\lambda, G) = [\Phi Z(G)]^B \lambda^{-A}$$

with

$$Z(G) = [1 - \left(\frac{G-280}{G-280}\right)^{1/\eta}]^\eta$$

Here, $Z(G)$ is a damage function adopted from a supply-driven climate model constructed by Rezai, Foley, and Taylor (2012) with $G$ as an observed level of CO$_2$ concentration and $\bar{G}$ as a level at which extremely severe climate damage occurs. The shape of $Z(G)$ as $G$ varies between a pre-industrial level of 280 ppmv and $\bar{G}$ depends on the parameter $\eta$. With $\bar{G} = 780$, $Z(G)$ is plotted for different values of $\eta$ in Figure 2.

**Figure 2**

With the numbers from the appendix, the output multiplier from the joint $u(\pi)$ and $\pi(u)$ system turns out to be 1.7046. The elasticity of $X$ with respect to $K$ is 0.75. Across business cycles the output/capital ratio is stable in the USA but has been falling for several decades in rapidly growing developing economies, so an elasticity of 0.75 is not out of line. It is also consistent with values quoted in debates about (neoclassical) growth model convergence, e.g. Mankiw, Romer, and Weil (1992).

*The Role of Productivity*

Mainstream growth models of climate change usually assume that labor productivity $\xi$ is exogenous. But in the Keynesian tradition productivity has long been
treated as endogenous in the macro system. Because productivity growth is often discussed, the model is set up in terms of a differential equation for $\dot{\xi} = d\xi/dt$.

An ostinato theme in ecological economics is that increasing energy use plays a key role in supporting labor productivity growth. To illustrate, let $\varepsilon = X/E$ stand for energy productivity. If $e = E/L$ is energy intensity then a useful identity follows immediately,

$$e = \xi/\varepsilon.$$  

(7)

Available data can be used to illustrate this accounting. There appears to be a robust relationship between increasing energy use per worker and labor productivity (Taylor, 2009), with an elasticity possibly exceeding one (meaning that there is a positive relationship between energy intensity and energy productivity). These observations are consistent with the view expressed by Smil (2005) and many others that much productivity-increasing technical change relies on higher energy use per unit employment.

Other factors may also influence productivity growth. Kaldor (1957, 1978) worked with a “technical progress function.” The basic idea is that greater capital stock and/or faster output growth will permit production to take place with decreasing costs and also allow more advanced technologies to be brought into play. For present purposes, $\dot{\xi}$ can be treated as an increasing function of $g$. As discussed in connection with Figure 1, higher productivity will increase profits and investment so that $\partial \xi / \partial \xi > 0$ through the Kaldor channel.

6 This specification is a differential version of Kaldor’s (1957) “Mark II” technical progress function.
Another line of thought suggests that output may not be strongly curtailed by a reduction in employment, basically because productivity goes up. In the US in 1930, this idea justified an apparently successful cut in the length of the working day by the Cornflakes tycoon W. K. Kellogg. Advanced by Sen (1966), it showed up in debates during the 1960s about surplus labor in developing economies – withdrawing labor from a “subsistence” sector was supposed to reduce the level of production by very little. The implication is that \( \dot{\xi} \) may be a decreasing function of \( \lambda \). Because \( \lambda = \kappa u / \xi \) from (4), this linkage would induce a positive feedback from \( \xi \) into its own growth. To emphasize the impact of the productivity shift on employment, however, one can set up a specification in which \( \partial \dot{\xi} / \partial \xi < 0 \).

Finally, higher GHG concentration can either stimulate or cut directly into productivity growth.

\[
\dot{\xi} = \xi \{ Qg - R\lambda + \left( 1 + \frac{S}{z(G)} \right) T\delta \}
\]

(8)

with \( \varepsilon \) to be determined by \( \xi \) and \( \varepsilon \) from (7).\(^7\)

The Kaldor effect shows up via \( \kappa \), with \( \partial \dot{\xi} / \partial g > 0 \). The Kellogg-Sen productivity link means that \( \partial \dot{\xi} / \partial \lambda < 0 \) (when \( R > 0 \)). The energy linkage is \( \partial \dot{\xi} / \partial \delta > 0 \), with energy productivity \( \varepsilon \) following from (7). With a positive value of \( S \) the GHG impact, based on the damage function (6) already used in (5), means that \( \partial \dot{\xi} / \partial G > 0 \). The response goes the other way when \( S < 0 \).

It is assumed below that \( \varepsilon \) converges to a stable level over time. The implication from (8) is that as energy intensity saturates \( \xi \) will converge to a constant value unless

\(^7\) An alternative specification, perhaps more appropriate under conditions of severe climate stress, would be to set up a differential equation for \( \varepsilon \) and treat \( \xi \) as the accommodating variable. For reasons of space this option is not pursued here.
independent of \( \kappa \) and \( G \). Both \( \xi \) and \( e \) will affect “transient” behavior of the model before they reach (nearly) constant values.

**Capital Stock Growth**

The standard convention in economic growth theory is that models should be set up so that all relevant variables converge toward a “steady state” at which they increase at the same exponential rate (perhaps equal to zero in “stationary state”). The present model is no exception.

All the responses discussed above will influence the details of convergence toward (or divergence from) a steady state. To bring in dynamics explicitly, let \( \dot{k} = \frac{dk}{dt} \) so that \( \ddot{k} = \frac{\dot{k}}{k} \) is the growth rate of the capital/population ratio. If \( \delta \) is the depreciation rate of capital and \( n \) is the rate of population growth (\( \dot{N} = n \)) then the increase in \( k \) over time is

\[
\dot{k} = \kappa (g - \delta - n) .
\]  

(9)

If \( n \) is constant (at least in some long run), then even though population growth may not be equal to zero (9) is independent of time and amenable to analysis of stability. It has a steady state solution with \( \dot{k} = 0 \),

\[
g = \delta + n \]

(10)

and from (1) we get

\[
su = \delta + n + (h - \tau u) + mu .
\]

(11)

In the simulations below, the depreciation coefficient \( \delta \) is set to 0.05 (corresponding to a 20-year lifetime for capital goods). If the population growth rate were zero, then saving would only have to pay for depreciation, the fiscal deficit, and mitigation. The share of the fiscal deficit in output is set to 0.03. The share of mitigation
spending might be 0.0125. If \( u = 0.3 \) then from (11) we have \( 0.3s = 0.05 + 0.3(0.03 + 0.0125) \) or \( s = 0.2092 \) in steady state, below the initial level of 0.24 postulated in the appendix. If and when the world economy reaches a steady state with zero population growth, a more modest saving effort would be required. Implications for the doctrine of “sustainable consumption” are discussed below.

Equation (9) is locally stable because a higher level of \( \kappa \) reduces the profit share and thereby the investment/capital ratio \( g \). The implication is that \( \partial \kappa / \partial \kappa < 0 \), or the standard stability condition applies. A higher level of \( G \) also reduces profitability, so that \( \partial \kappa / \partial G < 0 \). If there were no GHG feedback, the dark solid curve at the top of Figure 3 shows that capital per capita would rise steadily over time to approach a steady state. (All the other curves are discussed below.)

**Figure 3**

*Energy Use and GHG Accumulation*

The next step is to set up accounting describing how GHG accumulation and global warming are driven by the use of energy in production, giving rise to a two-dimensional dynamical system in \( \kappa = K/N \) and \( G \). With both variables at a steady state, \( K \) would be growing at the population growth rate \( n \). Final stabilization or reduction of GHG concentration would require \( n \leq 0 \).

---

8 Some demand-driven models assume an investment function in which \( g \) adjusts to drive \( u \) to a target level \( \tilde{u} \) — a strong constraint on the steady state. As shown in Figure 3 \( \kappa \) converges over centuries so that imposing such “consistency” on the investment function does not make much sense. When Henry Ford at age 50 set up his first Model T assembly line in 1913 it is hard to imagine that he was concerned with his fledgling company’s output/capital ratio 100 years thence.
GHG dynamics can be described by an expression similar to the “Kaya identity” from climate science (Waggoner and Ausubel, 2002),

\[ \dot{G} = \chi E - \mu(m)X - \omega G = [(\chi/\varepsilon) - \mu(m)]X - \omega G = vX - \omega G \]  \hspace{1cm} (12)

in which CO₂ emission goes up with energy use according to the coefficient \( \chi = \dot{G}/E \).

The function \( \mu(m) \) gauges the effectiveness of mitigation in reducing emission, and the \(-\omega G\) term captures slow natural dissipation of atmospheric CO₂. By reducing the profit share a higher level of \( G \) will decrease \( X \) so that \( \partial \dot{G} / \partial G < 0 \) when \( \nu > 0 \). A higher level of capital stock increases \( X \), making \( \partial \dot{G} / \partial \kappa > 0 \). Dividing both sides of (12) by \( G \) gives

\[ \dot{G} = [(\chi/\varepsilon) - \mu(m)]\Gamma - \omega = v\Gamma - \omega \]  \hspace{1cm} (13)

in which \( \Gamma = X/G \). The steady state solution to (12) is

\[ \Gamma = \omega/\nu \] .

To bring mitigation into the picture, we adopt the specification used in Rezai, Foley, and Taylor (2012)

\[ \mu(m) = \psi \frac{1-e^{-\phi m}}{\phi} \] . \hspace{1cm} (14)

This formula reflects decreasing effectiveness of expenditure on mitigation as its level increases. Figure 4 illustrates the shape of (14) for \( \phi = 6 \).

Figure 4

The simulations below suggest that significant reduction in emissions appears to be possible at the cost of 1.25% of world GDP. The total current outlay would be on the order of $750 billion per year, or more than 40% of worldwide military spending. Political implications are beyond the scope of this paper.
Dynamics

Table 1 shows numerical values in the base year for the partial derivatives of the three-dimensional dynamical system in $\kappa$, $G$, and $\xi$. For the moment, concentrate on the $2 \times 2 (\kappa, G)$ subsystem in the northwest corner. The opposing signs and relatively large magnitudes of the off-diagonal elements suggest that cyclicality is possible, with $\kappa$ as a leading or "prey" variable and $G$ as a follower or "predator." In the BAU simulation discussed below, the relatively strong response of $\dot{G}$ to $\dot{k}$ initially dominates the dynamics. As the simulation proceeds the magnitude of the negative partial derivative $\partial \dot{k} / \partial G$ increases. Eventually $G$ becomes big enough to force a sharp decline in $\kappa$ as $\dot{k}$ switches its sign from positive to negative.

Table 1

Rigorous analysis of cycles is of course only possible in the vicinity of steady state solutions, which will not apply when trending variables are included in the model. The two important ones are population and energy intensity. Both are assumed to approach stationary values eventually.

Population growth takes the form of a logistic

$$\dot{N} = 0.015 \left(1 - \frac{N}{10}\right),$$

reaching a maximum of 10 billion people from $N = 7$ initially.

Growth of energy intensity is similar. It ultimately increases by 50% from its current value of 4 to 6,

$$\dot{e} = 0.02 \left(1 - \frac{e}{6}\right).$$
As discussed above, labor productivity growth $\dot{\xi}$ goes toward zero along with $\dot{e}$.

**Simulations**

The differential equations (9), (12), (15), and (16) were solved numerically over 500 years using Mathematica. We first discuss the BAU solution and then how mitigation can set up a “reference path” or “mitigated solution.” The next topics are responses of this solution to slower population growth, a change in the mix of fossil fuels, slower growth of energy intensity, effects of GHG concentration on productivity growth, faster depreciation of capital, different mitigation costs, and “front-loading” of mitigation. Finally, specific points raised in the ecological and post-Keynesian economics literatures are discussed: “sustainable consumption” in the form of (initially) balanced budget mitigation, a possible “rebound effect” of mitigation spending on output and GHG accumulation, and Kaldor and Kellogg-Sen effects on productivity growth.

**BAU Simulation**

In Figure 3, a climate crisis in a BAU simulation starts to show up three or four decades into the simulation. The dark dashed curve falls below the capital per capita trajectory that would be observed without climate change. In Figure 5, $\kappa$ can be seen to grow by about 0.35% per year over 80 years, and then drop equally rapidly, reaching a minimum below 20 in year 200. It then settles down to a steady state level after population and productivity near their maximum values in equations (15) and (16). Post-crisis reductions in the employment/population ratio $\lambda$, output $X$, capacity utilization $u$, and consumption per capita $C/N$ are as dramatic as the fall in capital. The profit share and real wage adjust to support a lower investment/capital ratio $g$ in the steady state.

**Figure 5**
The climate crisis does not abate because atmospheric CO₂ concentration approaches 650 ppmv late in the century, and continues to rise for another century or so before leveling off above 700 ppmv. Cyclical dynamics dominate the trajectories until they reach steady state levels after population and productivity stop growing. It is of course possible to adjust parameters to support faster growth of κ coming out of the base year, but then the simulation crashes as κ fails to “turn the corner” at the peak of its period of growth.

*Mitigated Solution*

As illustrated by the dotted curve in Figure 3, mitigation of emissions is one way to avoid the catastrophe just sketched. In more detail, Figure 6 shows the effects of spending 1% or 1.25% of GDP on mitigation, slowing the growth of $G$ in equation (12). Especially with $m = 0.0125$, stable trajectories for macroeconomic variables are observed while CO₂ concentration remains in the range of 400-500 ppmv. Capital stock per capita continues to grow through two centuries, before declining slightly to a steady state level. The employment ratio rises, as do output and per capita consumption.

In effect mitigation induces a change in the model’s qualitative dynamics. Without mitigation the macroeconomy “overshoots” the growth path that would be feasible as GHG concentration goes up, then collapses and converges to a low level steady state. Setting mitigation spending to one percent of GDP almost reverses this dynamics, and 1.25% completes the job. These numbers of course depend on the details of the parameters, but the lesson is that a potential climate crisis can be averted by timely intervention (a point to be elaborated below).
Although 1.25% of output appears to be small, it amounts to $750 billion in the base year. This figure can be compared to total world defense spending, which is on the order of $1.75 trillion ($680 billion in the USA). Seriously combating global warming by mitigation of emissions would require very substantial mobilization of new resources.

In any case, we can take the solution with 1.25% mitigation as a reference path, for comparison with other changes. As shown in Figure 3, it moves the trajectory for $\kappa$ close to the solution unconstrained by $G$.

**Change in Fossil Fuel Mix**

In equation (12) GHG accumulation depends on the coefficient $\nu = (\chi / \varepsilon) - \mu(m)$. Besides increasing mitigation outlay $m$, decreasing the ratio $\chi / \varepsilon$ can be another way to reduce $\nu$. Figure 7 shows the effects of a reduction in the emissions/output ratio $\chi$ due to a shift in the mix of fossil fuels in use away from coal and oil toward natural gas. There is a modest reduction in CO$_2$ accumulation which permits better economic performance but the basic BAU oscillation persists.

**Figure 7**

**Slower Growth of Energy Intensity**

A change in the growth rate of energy intensity influences the model through at least two channels. If $\hat{\varepsilon}$ goes down, $\hat{\varepsilon}$ or the growth rate of energy productivity will decrease.

---

9 Optimal growth climate models use mitigation or a similar instrument to produce growth paths similar to the 1.25% solution here. In so doing, however, they divert attention from the intrinsically difficult dynamics of the $(\kappa, G)$ system.
decrease as well. Then GHG concentration should grow faster because the ratio $\chi / \varepsilon$ will fall less rapidly in (12). On the other hand, labor productivity growth $\xi$ will also drop, cutting into capital accumulation. Figure 8 shows that the latter effect dominates. Economic performance deteriorates, marginally slowing the rise in greenhouse gas.

**Figure 8**

*Slower Population Growth*

Population growth is often seen as a major force driving rising output and GHG concentration. From a supply-side perspective, this argument rests on the implicit assumption that there is full employment of the (working age) population. If employment is driven by aggregate demand and productivity, this argument becomes less compelling, as illustrated in Figure 9, based on the mitigated solution. The employment ratio increases a bit with slower population growth, and there is a modest reduction in GHG concentration. Overall, however, the reference path is not shifted significantly.

**Figure 9**

*Productivity Growth*

In contrast to population, productivity growth has strong effects on the solutions. Indeed, Table 1 shows that at least in the base year both the $(\kappa, \xi)$ and $(\tilde{G}, \xi)$ dynamical subsystems are potentially unstable, with negative determinants (the latter also has a positive trace).

As discussed above, the direct response of $\dot{\xi}$ to $\dot{G}$ can go either way. In a (perhaps overly) optimistic scenario, higher GHG concentration increases productivity
growth when $S > 0$ in (8). Figure 10 shows that the capital stock, employment, and output all increase more rapidly while CO$_2$ concentration declines. The decrease occurs because in equation (12) $\dot{G}$ is proportional to $X/\varepsilon$. Figure 8 shows that energy productivity rises more than in proportion to $\varepsilon$, reducing $\dot{G}$. This outcome of course depends on parameters, but it does illustrate the important role of productivity in the transient behavior of the system.

**Figure 10**

This pleasant situation reverses when $S < 0$ in (8). Declines in energy and labor productivity serve as a significant drag on the mitigated solution.

**Faster Depreciation**

As observed above, GHG accumulation can cause destruction of capital with adverse implications for growth. To illustrate their significance, Figure 11 shows the effects of 6% and 7% annual rates of depreciation (up from 5%) on the reference path. The results follow directly from the high elasticity of output with respect to capital that is built into the model. The higher depreciation rate so constrains output that CO$_2$ concentration actually goes down.

**Figure 11**

**Mitigation Costs**

The effectiveness of mitigation will of course depend on its cost. Figure 12 shows simulations with costs 15% above and below the level in the reference solution. Mitigation at higher cost permits more rapid growth of GHG concentration, although it
still prevents a climate crisis. Capital stock per capita, output, and (especially) employment all suffer. These outcomes reverse when cost is lower.

Figure 12

**Front-loaded Mitigation**

Intuitively, relatively high initial spending on mitigation will provide early benefits in the form of reduced CO₂ accumulation which will persist over time. Figure 13 presents an exaggerated pattern of front-loading, with $m$ declining from an initial level of 0.03 to 0.0125 over 500 years.

Figure 13

Figure 14 shows the outcome. Greenhouse gas concentration is substantially reduced while output and employment gain. These results suggest that partially postponing mitigation along a “climate policy ramp” (Nordhaus, 2008) may not be wise, as argued more fully by Rezai (2010). Timely intervention to avert a climate crisis is of the essence.

Figure 14

**“Balanced Budget” Mitigation**

As already mentioned, sustainable consumption is a recurring theme in ecological economics. Sustainability in this sense implies that the growth rate of consumption per capita should be low or negative (to be complemented by restructuring
the consumption basket in favor of less energy-intensive goods). We have already seen that a relatively low rate of saving may be required in a zero population growth rate steady state but what about the medium run?

A natural way to model sustainability is to assume that higher mitigation spending will be met by more public saving resulting from higher taxes. In other words, if $m$ increases by 0.0125, then the tax rate $\tau$ should rise by that amount as well. (Further effects through the system may make the extra tax take diverge from 1.25% of output, so the exercise is only “balanced budget” ex ante.)

Figure 15 suggests that the fiscal drag from higher taxes to support mitigation is visible but not large. Employment, output, and per capita consumption suffer, but not by very much. Greenhouse gas concentration decreases slightly because of the contractionary effect of higher taxes.

**Figure 15**

*Rebound Effect*

Similar observations apply to the “rebound effect.” The story is that greater spending on mitigation will stimulate aggregate demand, with high output creating a partial offset to the reduction in GHG accumulation resulting from higher $m$. Taking the levels of the change in concentration $\dot{C}$ along the BAU path as points of reference, Figure 16 shows that the extra emission caused by 1.25% mitigation (green dashed line) is trivial in comparison to the reduction in $\dot{C}$ illustrated by the comparison levels of $\dot{C}$ along the BAU and mitigated trajectories.

**Figure 16**
**Kaldor and Kellogg-Sen Effects**

In the differential specification of Kaldor’s (1957) earlier version of the technical progress function in equation (8), \( Q = 1.0 \) generates a modest positive feedback of \( g \) into \( \xi \), with results shown in Figure 17. Because energy productivity \( \varepsilon \) rises along with \( \xi \), GHG concentration falls. The potentially destabilizing feedbacks between \( \xi \) on the hand and \( \kappa \) and \( G \) on the other (noted above) dominate the responses.

**Figure 17**

The scenario reverses when the Kellogg-Sen effect is brought with \( R = 0.5 \). In particular there is a pronounced decrease in the employment ratio \( \lambda \). The real wage also falls. This simulation is of interest insofar as it sheds light on proposals to cut back on the increase in \( G \) by reducing employment via a shorter workweek, etc. (Schor, 2010). Given the macro linkages among variables in the model, reduced employment would have to be accompanied by higher productivity.

**Neoclassical Alternative**

Given the importance of cyclicality between \( \kappa \) and \( G \), it makes sense to ask if it is an artifact of the demand-driven specification. Figure 3 shows that such is not the case. It presents simulations with a Solow-Swan growth model. We first briefly describe the specification, and then results.

There is supposed to be a neoclassical aggregate production function (Cobb-Douglas for simplicity)

\[
X = Z(G)A(\xi \lambda N)^{0.6}K^{0.4}
\]
with $A$ as a calibration parameter. The employment/population ratio $\lambda$ is no longer an adjusting variable but stays fixed at its base year level (a full employment assumption) and the damage function $Z(G)$ is the same as in the demand-driven version. If $v = X/N$ this equation becomes

$$v = Z(G)A(\xi\lambda)^{0.6}\kappa^{0.4}.$$

Replacing (9) the growth equation for $\kappa$ is

$$\dot{\kappa} = (s + \tau - m)v - (h + n + \delta)\kappa$$

which can be solved using the base year values of $\lambda$ and $G$ along with (15) and (16) for $\dot{\xi}$ and $\dot{N}$.\(^{10}\)

In Figure 3, the orange solid line gives the unconstrained solution to (17). If it is then solved jointly with (12) for $\dot{\xi}$, the lower solid blue curve results. The full employment assumption partially stabilizes the solution in comparison to the demand-driven version. Also, $\chi$ and therefore new GHG emission are less sensitive to $K$ in the Cobb-Douglas formulation. Nevertheless the cyclical rise and fall of $\kappa$ going into the steady state persists. Finally, 1.25% mitigation returns the simulation close to its unconstrained variant.

More detailed mitigation simulations give outcomes with the same general patterns as shown in Figure 6.

**Appendix: Data and Parameterization**

For equations (1)-(3), world output or GDP is set roughly $X = 60$ (trillion dollars). With $K = 200$, $u = 0.3$. With the share of government spending in output $H/X = 0.33$, $h = 0.099$. If the fiscal deficit is normally three percent of GDP, then $\tau = 0.3$. A plausible

\(^{10}\) Textbook presentations set $\kappa = K/\xi L$ but we avoid this specification to maintain comparability with the demand-driven model.
level of the world saving rate is $s(\pi) = 0.24$. If $m = 0$ initially, then the
investment/capital ratio becomes $g = 0.063$. The profit share of output is roughly $\pi = 0.4$. If the saving rate from profits is $s_\pi = 0.28$ then the rate from wage income becomes $s_w = 0.2133$. With $\alpha = 0.25$, $g_0 = 0.0033$. supports the investment function. Aggregate
demand will be profit-led if $\alpha + s_w - s_\pi > 0$, a condition satisfied by these numbers.

For use in (4) - (6), we set world employment $L = 3$ (billion) and population $N = 7$
so that $\lambda = 0.42857$. With current output of 60, $\xi = 20$. The capital/population ratio is
$\kappa = 200/7 = 28.5714$ (or $28,571$ per capita) After experimentation with sensitivities we
set $\eta = 0.5$, $A = 2$, and $B = 2$. The parameter $\Phi$ was “calibrated” to fit $\pi = 0.4$, taking the
value of 0.2792.

For initial simulation runs with (8) we set $Q = R = S = 0$, and $T = 1.5$. The
constant $J$ becomes equal to 1.

In (13), it is simplest to think of energy use in terms of terawatts of power (as
opposed to exajoules of energy per year). The current world level is about 15 terawatts,
of which 12 are provided by fossil fuels. Fossil fuel energy productivity becomes
$\varepsilon = 60/12 = 5$. This energy use generates about 7 gigatons of carbon emissions per
year, corresponding to an increase in $G$ of 3.37 ppmv. The observed increase is about 2
ppmv, so that $\hat{G} = 2/400 = 0.005$ with atmospheric dissipation of 1.37 ppmv. The
dissipation coefficient becomes $\omega = 1.37/400 = 0.0034$.

Assuming that there is now no effective mitigation or $\mu(m) = 0$ (12) becomes
$$\hat{G} = (\chi/\varepsilon) \Gamma - \omega.$$ 
With emissions of 3.37 and fossil fuel energy use of 12, the ratio $\chi = 3.37/12 = 0.2808$
and $\nu = \chi/\varepsilon = 0.0562$. The balance equation for $\hat{G}$ works out to be
\[ 0.005 \approx (0.0562)(0.15) - 0.0034 = 0.0084 - 0.0034. \]

In (14) if one percent of output (or $0.6 trillion) is devoted to mitigation \((m = 0.01)\) we have

\[ \frac{1-e^{-0.06}}{6} = 0.0097. \]

That is, the cost-effective outlay is \((0.6)(0.97) = 0.582\).

For use in (12) and (14), a fairly high estimate of the cost of removing one ton of carbon emissions is $160, or $44 per ton of CO\(_2\) (roughly twice the level now being considered by the government in the USA, according to Ackerman and Stanton, 2012). To reduce atmospheric CO\(_2\) concentration by 1.0 ppmv would require removal of 2.07 gigatons of carbon from emissions at a total cost of \((2.07)(\$0.16\ trillion) = \$0.331\ trillion\). Spending one percent of output would mitigate \(0.582/0.331 = 1.7583\ ppmv\). So we get the change in emissions as \(-\Delta \dot{G} = 1.7583 = 0.582\psi\) or \(\psi = 3.0211\).
References


<table>
<thead>
<tr>
<th></th>
<th>$\kappa$</th>
<th>$\dot{G}$</th>
<th>$\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ddot{\kappa}$</td>
<td>$-0.74$</td>
<td>$-0.00102$</td>
<td>$0.1$</td>
</tr>
<tr>
<td>$\dot{G}$</td>
<td>$0.07862$</td>
<td>$-0.00397$</td>
<td>$0.05616$</td>
</tr>
<tr>
<td>$\ddot{\xi}$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0.03$</td>
</tr>
</tbody>
</table>

Table 1: Numerical base year Jacobian matrix for $\kappa$, $\dot{G}$, and $\ddot{\xi}$. 
Figure 1: Short-run $u(\pi)$ and $\pi(u)$ schedules based on model parameters.

Figure 2: Damage function for the profit share.
Figure 3: Dynamics of $\kappa$ for demand-driven and neoclassical growth specifications.
Figure 4: Cost-effectiveness of mitigation.
Figure 5: Detailed results from BAU simulation.
Figure 6: Responses of BAU simulation to different levels of mitigation.
Figure 7: Responses of BAU simulation to shifts in fossil fuel mix.
Figure 8: Responses of BAU simulation to slower growth of energy intensity.
Figure 9: Effects of different rates of population growth on the reference path.
Figure 10: Trajectories of variables when $\dot{x}$ responds to $G$. 
Figure 11: Responses of trajectories to faster depreciation of capital.
Figure 12: Effects on solutions of higher and lower mitigation costs.
Figure 13: Front-loading of mitigation.
Figure 14: Effects of front-loaded mitigation.
Figure 15: "Balanced budget" mitigation.
Figure 16: The “rebound effect” of mitigation spending on BAU emission growth.
Figure 17: Kaldor and Kellogg-Sen effects